# Maths Modelling Challenge: Years 7 - 12

Teacher Handout

## Scenario: The Mathematics of Kindness

From both a mathematical and social standpoint, the way in which a group of people can interact with each other is very large indeed. A small one-off interaction can have far reaching consequences because they influence the follow-on behaviour of all who experience them.

Studies have shown that random acts of kindness not only benefit the recipient but also the giver. This limitless and "transmissible" nature of human interaction means that simple acts of kindness have the potential to make our world a better place, providing people with a more meaningful sense of connection to their community. In fact, human interactions have similar properties to a virus outbreak. One person can interact with many, a super spreader if you will – the spread and growth rate is exponential and receiving an act of kindness effects how you then behave – kindness is contagious!

The World Health Organisation (WHO) has decided to model the impact of random acts of kindness around the world as part of a mental health initiative. WHO has asked your group to help them model the local and global impact of small acts of kindness.

### Key points the WHO have asked you to address are:

- If all members of your group each decided to carry out one random act of kindness for a different person every day, and assuming these people (receiving the act of kindness) then did the same thing from then on and passed this on, and then the next receivers of an act of kindness then did the same thing, etc... how long would it take for everyone in your school to receive an act of kindness?
- How many acts of kindness would you have to carry out before everyone in the world received one?
- How would you need to adjust your model to reflect more realistic interactions and social connections and what effect would this have on your projected timeline?

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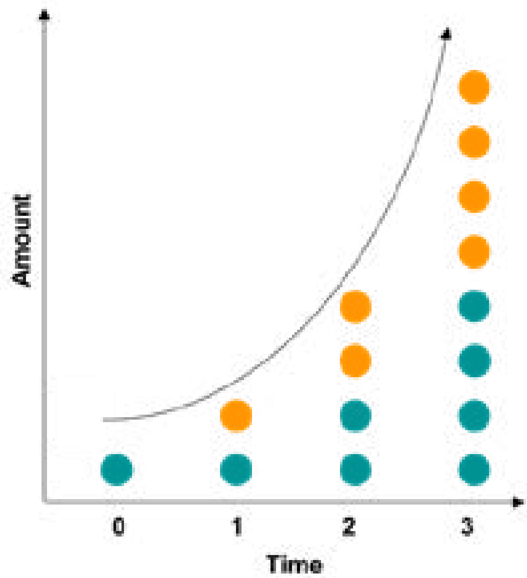
## SHORT INTRODUCTION ON MODELLING GROWTH RATE

### Modelling growth rate

Continually growing systems are all around us, from our growing population, to interest earnt on our savings accounts, to cells dividing and multiplying in our own bodies. We can use mathematics to model this growth. COVID-19 has brought some of the mathematical terminology we use to describe growth rate into everyday conversation. You may have heard of the term “exponential growth”. This is a term used to describe and system that is continually growing at a common rate. The spread of viruses like COVID-19, when left to their own devices will spread continuously (their infection rate) at a common rate (how contagious they are). But what do we actually mean by exponential growth? And how is it used to model the spread and growth rate of a system or virus?

### Exponential growth

Let’s imagine a really simple system that always doubles after a period of time. For example, imagine a bacteria splitting in 2 periodically. If we were to graph that system, it would look like this.




Mathematically, if we wait a period of time so we have x splits then we get $2^x$ times as much stuff than when we started. With 1 split (time point 1 on the graph) we have $2^1$ or 2 times as much. With 2 splits (time point 2 on the graph) we have $2^2$= 2x2=4 times as much stuff. With 4 splits we have $2^4$= 2x2x2x2=16 times as much and so on. The general formula we can use to model this pattern is:

growth = $2^x$

where “x” is known as the exponent.

If we take this a step further we model all sorts of growth, not just doubling but say tripling. Doubling is 100% growth. So, we can rewrite our formula like this:

growth = $(1 + 100\%)^x$

It’s the same equation, but we separate 2 into what it really is: the original value (1) plus 100%. Of course, we can substitute any number (50%, 25%, 200%) for 100% and get the growth formula for that new rate.

The above is covered in more detail here.

[https://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/](https://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/)

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### How is this useful?

In many cases we know the growth rate of a system. Take COVID-19, after initial observations scientists can evaluate the spread or growth rate of the virus. Once they know this they can then work out how many days "x" it will take to infect a certain number of people and therefore understand the impact of the virus and how much time we have to react. So, the above becomes:

Number of people infected = $(1 + \text{rate\%})^{\text{number of days}}$

We can work out the number of days "x" it will take to infect a certain number of people by using logarithms "log". Logarithms can be thought of as a transform function, allowing us to find x when we know growth and rate. There are some really great log conversion calculators online at https://www.rapidtables.com/calc/math/Log_Calculator.html

So:

Number of people infected = N = $(1 + \text{rate\%})^x$

Number of days to infect N number of people = x = log(N)/log(1 + rate%)

So how many days would it take for our bacteria above to split into 32 bacteria if they split once every day? Well if we use the above equations, we know that:

$32 = (1 + 1)^x$

therefore:

x = log(32)/log(2) = 5/1 = 5 days!

You can check that this is the case by doing 2x2x2x2x2 = 32!

Now you have the tools to be able to simply model and more importantly predict growing systems. What if the rate tripled or halved?

## USEFUL RESOURCES

There are many ways to approach this problem and many sources for reference. Below is a list of useful links and hints that provide some background reading and may aid in your approach to the problem. These can all be access without special licenses to journals.

Useful links around the idea of using mathematics to model kindness.

https://betterexplained.com/articles/an-intuitive-guide-to-exponential-functions-e/

https://plus.maths.org/content/mathematics-kindness

https://www.honeyfoundation.org/the-mathematics-of-kindness/

## INDEX

*Contagion rate*: the spreading of an idea, behaviour or disease.

*Exponential growth*: growth whose rate becomes ever more rapid in proportion to the growing total number or size.

*Growth rate*: this tells you how quickly a population or system increases in size.

*Transmission*: the transfer of an idea, disease or behavior from one person to another.

*Transmissible*: the ability or trait of a disease, or in this cause behaviour, to be passed on from one person or organism to another.

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## Additional Notes for Teachers

All entries must be students' own work and verified on submission. Acceptable teacher support for students please refer to below *Footnotes - points of discussion and exploration*.

- Students will need to research exponential and logarithmic functions to evaluate the exponent of growth required to reach the number of students in their school and population of the world. We have provided a short introduction on modelling growth rate in the student's handout under "Useful links".
- In terms of adjusting the model to reflect more realistic social connections, encourage students to thinking about social networks as well as who might qualify as a "super spreaders" and what their role and impact might be.
- It is unlikely everyone passing on an act will reach out to a different person as we all form part of shared social groups. Fewer people will have connections outside of Australia so we can imagine that growth rate will decrease when passing from country to country. Furthermore, we may have drop out along the way, people may not pass it on. Calculating this is complex but encourage students to discuss the impact on their initial model rather than calculating it.

### Example Work Through

#### Exponential growth of kindness in school

Number of students and staff at school = **X** (for this example X = 1000*)
Number of people in my group = **N** (for this example N = 2)
Number of days required = **d**
$\mathbf{N^d = X}$
To find **d** we need to perform a log transform.**

**d**= ln(**X**)/ln(**N**) = 9.97 days = 10 days

#### Exponential growth of kindness to the rest of the world

Global population = **X** (for this example X = 7.7bn***)
Number of people in my group = **N** (for this example N = 2)
Number of days required = **d**
$\mathbf{N^d = X}$

**d**= ln(**X**)/ln(**N**) = 32.84 days = 33 days

The 2 people who started this initiative would have to carry this out for just over a month.****

#### Adjusting the model‡

1. A simple way to approach this could be to break it down into temporal sections.
2. Days to reach Australian population. Then assume only 1 in 100 Australians know someone overseas to calculate the next growth stage etc. Perhaps only 60% pass it on, or the initial participants may stop after a few weeks±
3. What are the role and impact of "super spreaders" people with larger social networks that perform more than 1 act of kindness a day?

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## Additional Notes for Teachers

### Footnotes - Points of Discussion and Exploration

* Encourage students to discuss the difference in days between school sizes to understand the mathematics behind exponential rather than linear growth.

** You may wish to run through a simple example of exponential growth and how we transform between e and ln at the beginning of this session.

*** This may change depending on time carried out, but this can be found online.

**** Encourage students to estimate beforehand and then compare this with their results.

‡ This will vary greatly from group to group but students should justify their working out and evaluation.

† This is a good opportunity to play around with exponent rules of division and multiplication to get an idea of how dropout will affect spread. This is very complex to actually model but encourage student to discuss how they might go about this and what mathematics they would use.

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